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# Tri-N-ification

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## Abstract

We consider a natural generalization of trinification to theories with $3N$ SU(3) gauge groups. These theories have a simple moose representation and a gauge boson spectrum that can be interpreted via the deconstruction of a 5D theory with unified symmetry broken on a boundary. Although the matter and Higgs sectors of the theory have no simple extra-dimensional analog, gauge unification retains features characteristic of the 5D theory. We determine possible assignments of the matter and Higgs fields to unified multiplets and present theories that are viable alternatives to minimal trinified GUTs.



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## I. INTRODUCTION

Trinification [1, 2, 3, 4, 5, 6] refers to unified theories based on the gauge group $G_T = \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$, with gauge coupling equality imposed at a high scale, typically by a discrete symmetry that cyclically permutes the gauge group labels $C$, $L$, and $R$. The Higgs fields responsible for breaking $G_T$ to the standard model gauge group appear in the **27**-dimensional representation,

$$\phi(\mathbf{27}) = \phi_{LR}(\mathbf{1}, \mathbf{3}, \bar{\mathbf{3}}) + \phi_{RC}(\bar{\mathbf{3}}, \mathbf{1}, \mathbf{3}) + \phi_{CL}(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \ . \tag{1.1}$$

The component fields $\phi_x$ transform as $(\mathbf{3},\ \overline{\mathbf{3}})$'s under the pair of gauge groups $x = LR$, $RC$, and $CL$, respectively. The theory thus specified has a simple moose representation, shown in Fig. 1, with the component fields $\phi_x$ serving as the 'links' that connect neighboring gauge group 'sites'. This structure is reminiscent of a deconstructed higher-dimensional theory [7, 8], aside from the fact that the link fields in Eq. (1.1) do not have the same purpose as in deconstruction, namely, to break chains of replicated gauge groups down to their diagonal subgroup.

Nevertheless, the structure of Fig. 2 suggests a natural generalization to trinified theories in which the group factors $C$, $L$, and $R$ are replicated $N$ times. We discuss this generalization in Section II. These theories also have a simple moose representation, and a gauge spectrum that can be interpreted via deconstruction. In particular, we will see that the gauge sector of the theory is a four-dimensional analog of the 5D trinified theory of Ref. [9], in which gauge symmetry breaking effects are localized entirely on a boundary. Such 5D theories have interesting properties [10], such as the logarithmic running of the gauge coupling difference $\alpha_i^{-1} - \alpha_j^{-1}$, for $i \neq j$, and a delay in the scale of unification above $2 \times 10^{16}$ GeV, the value obtained in the minimal supersymmetric standard model (MSSM). The construction of purely four-dimensional theories with such properties is clearly worthy of pursuit, and has lead to interesting results in the case of SU(5) [11, 12] and SO(10) [13] unification. The present work complements this body of literature by introducing a new class of 4D unified theories that are closely related to the deconstruction of 5D trinified GUTs.

That said, it will not be the purpose of this paper to present a literal deconstruction of the theories discussed in Ref. [9]. Rather, we proceed from a mostly 4D perspective and develop models that are viable and economical. For example, in determining the embedding

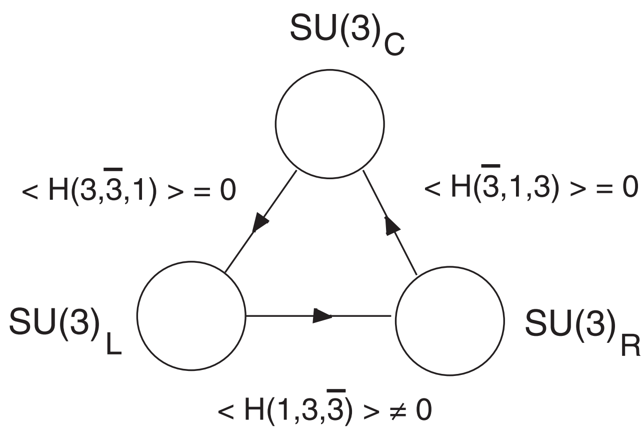


FIG. 1: Moose diagram for conventional trinified theories.

of matter fields in the theory, we don't follow the prescription of deconstruction at all, so that our 4D theory as a whole cannot be mapped to a local 5D theory in the continuum limit. The higher-dimensional flavor of gauge unification is nonetheless retained leading to a new and interesting class of 4D unified theories.

Our paper is organized as follows. In Section II, we review conventional trinification and define a generalization to $3N$ replicated SU(3) groups. In Section III, we study the gauge boson spectrum of the model, for arbitrary values of the localized symmetry-breaking vacuum expectation values (vevs). In Section IV, we apply these results to study gauge coupling unification in this class of models. In Section V, we describe how one may successfully include matter and light Higgs fields, so the low-energy particle content is the same as in the MSSM. In Section VI, we summarize our results and suggest directions for future study.

## II. TRINIFICATION GENERALIZED

Conventional trinified theories [1, 2, 3, 4, 5, 6] are based on the gauge group $G_T = \mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R \ltimes Z_3$, where $\ltimes$ indicates a semidirect product. The $Z_3$ symmetry cyclically permutes the gauge group labels $C$, $L$ and $R$, ensuring a single unified coupling at the GUT scale. The breaking of $G_T$ to the standard model gauge group requires a Higgs field in the **27**-dimensional representation,

$$\phi(\mathbf{27}) = \phi_{LR}(\mathbf{1},\mathbf{3},\bar{\mathbf{3}}) + \phi_{RC}(\bar{\mathbf{3}},\mathbf{1},\mathbf{3}) + \phi_{CL}(\mathbf{3},\bar{\mathbf{3}},\mathbf{1}) \ , \tag{2.1}$$

where the numbers shown represent the $\mathrm{SU}(3)_C$, $\mathrm{SU}(3)_L$ and $\mathrm{SU}(3)_R$ representations, respectively. At least one such field must develop the vacuum expectation value,

$$\langle \phi_{LR}(\mathbf{1},\mathbf{3},\bar{\mathbf{3}})\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & v_2 & v_1 \end{pmatrix} \ . \tag{2.2}$$

The presence of vevs only in the third row assures that $\mathrm{SU}(2)_L$ remains unbroken, while nonvanishing 32 and 33 entries leave a single unbroken U(1) factor, generated by a linear combination of the diagonal generators of $\mathrm{SU}(3)_L$ and $\mathrm{SU}(3)_R$. Identifying this with hypercharge $Y$, one finds,

$$A_Y^\mu = -\frac{1}{\sqrt{5}}(A_L^8 + \sqrt{3}A_R^3 + A_R^8)^\mu \ . \tag{2.3}$$

Note that $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry allows one to rotate away the vev $v_2$ in Eq. (2.2). Therefore, it is usually assumed that at least two **27** Higgs fields with vacuum expectation values in the desired entries are present in the theory. We will return to this issue in Sec. V. One can verify that this construction yields the standard GUT-scale prediction for the weak mixing angle $\sin^2\theta_W = 3/8$.

Let us focus on the structure of the gauge and unified-symmetry-breaking sectors of the theory. A conventional trinified theory can be represented by the moose diagram shown in Fig. 1. For the sake of simplicity we display only a single Higgs **27**. The moose representation makes it clear that the **27** is anomaly-free, which is relevant for the Higgs representations since we assume supersymmetry. The $Z_3$ symmetry is encoded in the symmetry of the moose under rotations by 120°.

Now consider the generalization of this moose to $3N$ gauge groups, as shown in Fig. 2. Each link field transforms as a $(\bar{\mathbf{3}}_i, \mathbf{3}_{i+1})$ under consecutive SU(3) groups, reading around the moose diagram clockwise. We assume that a generic link has vev

$$\langle Q_x(\bar{\mathbf{3}}_j, \mathbf{3}_{j+1})\rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & v \end{pmatrix} , \tag{2.4}$$

where $x = C, L$ or $R$. This breaks a chain of SU(3) factors, namely $\mathrm{SU}(3)_{x_i}$ for $i = 1 \ldots N$, down to its diagonal subgroup. The diagonal subgroups are precisely

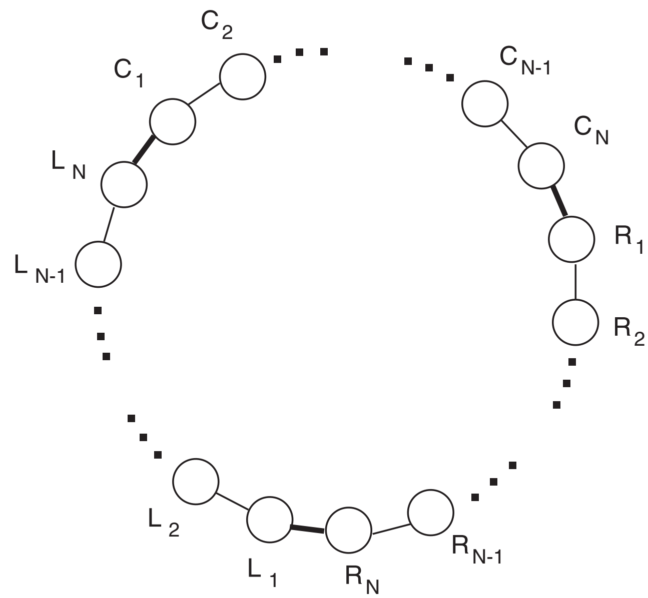


FIG. 2: Generalization to $3N$ gauge groups (arrows left implicit). The links transforming as $(\bar{\mathbf{3}}_N, \mathbf{1}, \mathbf{3}_1)$ and $(\mathbf{3}_1, \bar{\mathbf{3}}_N, \mathbf{1})$ have no vacuum expectation values, while the link $(\mathbf{1}, \mathbf{3}_1, \bar{\mathbf{3}}_N)$ has the vevs given in Eq. (2.2).

SU(3)$_C$×SU(3)$_L$×SU(3)$_R$ of conventional ($N = 1$) trinified theories. The three links which are not generic are identified with the links of the $N = 1$ theory and have the corresponding expectation values. In particular, the links $(\bar{\mathbf{3}}_N, \mathbf{1}, \mathbf{3}_1)$ and $(\mathbf{3}_1, \bar{\mathbf{3}}_N, \mathbf{1})$ have no vevs and serve to truncate a linear moose that contains unbroken SU(3)$_C$. If the link $(\mathbf{1}, \mathbf{3}_1, \bar{\mathbf{3}}_N)$ also had no vev, we could make an analogous statement for SU(3)$_L$ and SU(3)$_R$; we assume, however, that this link has precisely the expectation value necessary to break SU(3)$_L$×SU(3)$_R$ down to the electroweak gauge group of the standard model. Thus, the total effect of the link vevs is to break

$$SU(3)^{3N} \to SU(3)_C \times SU(2)_W \times U(1)_Y , \tag{2.5}$$

below the scales $v$, $v_1$ and $v_2$.

At this point, we have said nothing about the values of the $3N$ gauge couplings. To maintain gauge unification, we assume that the theory defined by Fig. 2 is restricted by a $Z_3$ symmetry that sets equal three sets of $N$ gauge couplings each,

$$g_{C_i} = g_{L_i} = g_{R_i} \qquad i = 1 \ldots N \, . \tag{2.6}$$

Thus, we see that the moose in Fig. 2 is also symmetric under rotations by 120°, like the

$N=1$ theory. Unification is maintained as in the $N=1$ theory since the couplings of the diagonal subgroups are given by

$$\frac{1}{g_x^2} = \sum_{i=1}^{N} \frac{1}{g_{x_i}^2} \tag{2.7}$$

for $x = C, L$ or $R$. Note that the vev of the $(\mathbf{1}, \mathbf{3}_1, \bar{\mathbf{3}}_N)$ link breaks both $\mathrm{SU}(3)^{3N}$ and the cyclic symmetry of the moose.

It is important to note that we are agnostic as to whether the $Z_3$ symmetry is a symmetry of the full theory, including the fermion representations, or is only an accidental symmetry of the gauge sector. The latter could be the case if, for example, all of the gauge couplings are equal at a high scale due to the dynamics of a more complete, high-energy theory. We will remain open to both possibilities in our subsequent model building.

In the theory we have described thus far, the special links between $L_1$-$R_N$, $C_1$-$L_N$ and $R_1$-$C_N$ contribute to the low-energy particle content of the theory. On the other hand, we wish only to have two light Higgs doublets together with the matter content of the MSSM. How we may arrange for this is discussed in Sec. V. We first, however, address the less model-dependent issue of gauge unification, assuming that the low-energy matter and Higgs content is that of the MSSM.

## III. GAUGE BOSON SPECTRUM

Let us begin by discussing the spectrum of the $SU(3)_{C_i}$ gauge multiplets, for $i = 1 \ldots N$. This portion of the theory is a linear moose with $N$ unbroken SU(3) factors. The contribution to the gauge boson mass matrix from a link spanning the $j$ and $(j+1)^{\mathrm{th}}$ site is given by

$$\mathcal{L}_{j,j+1} = v^2 g^2 \mathrm{Tr}[A_C^j A_C^j - 2 A_C^j A_C^{j+1} + A_C^{j+1} A_C^{j+1}], \tag{3.1}$$

(with $A_C \equiv A_C^a T^a$) leading to the $N \times N$ mass squared matrix

$$M_C^2 = v^2 g^2 \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{pmatrix} \tag{3.2}$$

Here we take all SU(3) gauge couplings to be equal, and adopt this simplifying assumption henceforth. The mass spectrum given by Eq. (3.2) is that of a deconstructed 5D SU(3) gauge theory, and is well known [11],

$$m_c = \frac{2}{a} \sin \frac{\ell \pi}{2N} \qquad \ell = 0, 1, 2, \dots , \tag{3.3}$$

where $a = (vg)^{-1}$ is the lattice spacing. The $L$ and $R$ sectors of the moose are more interesting due to the presence of the nontrivial vev at the $L_1$-$R_N$ link. Let us define the $i^{\text{th}}$ gauge field as the 16-component column vector

$$A_i = [A^1_{L_j}, \dots A^8_{L_j}, A^1_{R_i}, \dots A^8_{R_i}] \ , \quad j = N + 1 - i \ , \quad i = 1 \dots N \, . \tag{3.4}$$

If one were to excise the $LR$ sector of the circular moose in Fig. 2, and to bend it about the special link at $L_1$-$R_N$, one would obtain a linear moose with $N$ sites, corresponding to gauge fields $A_i$ in Eq. (3.4). The symmetry breaking effects of the special link are confined to the $N^{\text{th}}$ lattice site. Thus, we will be able to interpret our result as a deconstructed 5D theory with bulk $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ gauge symmetry broken at a boundary. Since the $L$-$L$ and $R$-$R$ link fields give identical contributions to the $\mathrm{SU}(3)_L$ and $\mathrm{SU}(3)_R$ gauge boson mass matrices, respectively, we may express the mass matrix for the $A_i$ as

$$M^2_{LR} = v^2 g^2 \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & & \\ & & -1 & 2 & -1 \\ & & & -1 & 1 \end{pmatrix} + \begin{pmatrix} 0 & & & & \\ & 0 & & & \\ & & \ddots & & \\ & & & 0 & \\ & & & & \Delta \end{pmatrix} \tag{3.5}$$

where each entry represents a $16 \times 16$ matrix in $LR$ space. Only the boundary contribution $\Delta$ has a nontrivial structure in this space – all others are proportional to an implicit identity matrix. The form of $\Delta$ is precisely that of the $LR$ gauge boson mass squared matrix in a conventional, $N = 1$ trinified theory. Let $u$ be the 16-dimensional unitary matrix which diagonalizes $\Delta$:

$$u^\dagger \Delta u = \Delta_{\text{diag}} \ . \tag{3.6}$$

Then the $(N,N)$ entry of Eq. (3.5) may be diagonalized without affecting any of the others by letting $M^2_{LR} \to U^\dagger M^2_{LR} U$, where $U$ is the unitary matrix

$$U = \begin{pmatrix} u & & & & \\ & u & & & \\ & & u & & \\ & & & \ddots & \\ & & & & u \end{pmatrix} . \tag{3.7}$$

Since each 16-dimensional sub-block is now-diagonalized, we end up with 16 decoupled mass matrices, corresponding to the eigenvalues $v^2 g^2 \eta$ of $\Delta$:

$$M^2_\eta = v^2 g^2 \begin{pmatrix} 1 & -1 & & & \\ -1 & 2 & -1 & & \\ & & \ddots & & \\ & & -1 & 2 & -1 \\ & & & -1 & 1+\eta \end{pmatrix} . \tag{3.8}$$

Four of the eigenvalues of $\Delta$ are zero, corresponding to the unbroken $\mathrm{SU}(2)_W \times \mathrm{U}(1)_Y$ gauge bosons, while the remaining 12 are superheavy. For the massless states ($\eta = 0$), Eq. (3.8) reduces to Eq. (3.2), as one would expect, and the mass spectrum is given by Eq. (3.3). The massive gauge fields are more interesting. In this case, $\eta$ is nonvanishing and is of the order $v_i^2/v^2 \geq 1$. We can find the mass spectrum in the $\eta \neq 0$ case by exploiting a mechanical analogy. Consider the system of masses $m$ and springs shown in Fig. 3a. The equation of motion for the $i^{\text{th}}$ block is given by

$$\frac{d^2 x_i}{dt^2} = -K_{ij} x_j \tag{3.9}$$

where $K_{ij}$ has precisely the form of Eq. (3.8), with the identifications $v^2 g^2 = k/m$ and $\eta = K_0/k$. The squared frequencies of the normal modes of this mechanical system are precisely the eigenvalues of $K_{ij}$. Notice that as $K_0$ is made large (*i.e.* the massive gauge bosons of the last site in the moose are decoupled), the $N^{\text{th}}$ block in our spring system effectively becomes a fixed wall. We then obtain an $(N-1) \times (N-1)$ mass matrix of the same form as Eq. (3.8) with $1 + \eta = 2$. This is exactly what we expect for the massive gauge modes when the $N^{\text{th}}$ site has a smaller gauge symmetry than the other sites in the moose (see for example, Ref. [11].)

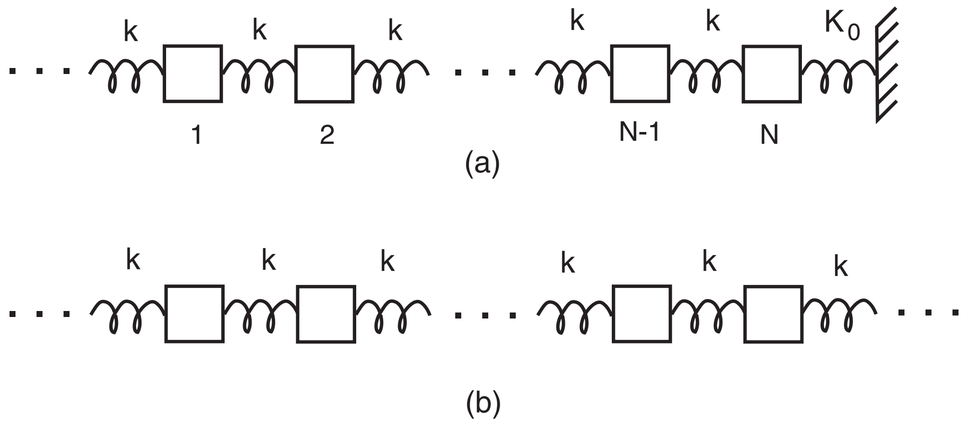


FIG. 3: (a) Mechanical analog for studying the gauge boson mass matrix. (b) Translationally invariant system.

The eigenvalues of $K_{ij}$ may be found by considering the translationally invariant system shown in Fig. 3b, and imposing boundary conditions that mimic the dynamics of the first and $N^{\text{th}}$ block of the system of interest. Let $\psi(a\,i)$ represent the displacement of the $i^{\text{th}}$ block about its equilibrium position, where $a$ is the inter-block spacing. The fact that the first block has no spring to the left is equivalent to the boundary condition

$$\psi(0) = \psi(a) \tag{3.10}$$

in the translationally invariant system. On the other hand, the effect of the spring with larger spring constant $K_0$ to the right of the $N^{\text{th}}$ block is replicated by the condition

$$\psi(a[N+1]) = -(\eta - 1)\,\psi(aN) \ . \tag{3.11}$$

In other words, in the infinite system, one requires that the $(N+1)^{\text{th}}$ block moves so that the force on the $N^{\text{th}}$ block is indistinguishable from that of a stiffer spring connected to a wall. We may thus consider a normal mode solution to the infinite system of the form

$$\psi \propto e^{i\,k\,x} + B\,e^{-i\,k\,x} \tag{3.12}$$

and determine the wave numbers $k$ allowed by these boundary conditions. We find that $N-1$ values of $k$ are determined by the transcendental equation

$$\cot(kaN)\cot(ka/2) = -(1 - 2/\eta) \tag{3.13}$$

leading to the eigenvalues

$$m^2 = \frac{4}{a^2}\sin^2\left(\frac{ak}{2}\right) \quad . \tag{3.14}$$

Here, $m^2$ represents the squared frequencies of the normal mode solutions in the mechanical system, and $N-1$ gauge boson squared masses in the problem of interest. The $N^{\rm th}$ gauge boson mass, however, is not given by Eqs. (3.13) and (3.14). The reason is that we have assumed that $k$ is real; complex $k$ is perfectly consistent with the translation invariance of the infinite system (see, for example, Ref. [15]). Taking $k = k_R + ik_I$, one finds another solution:

$$\begin{aligned} k_R = n\pi/a\,, \quad n \;&=\; \text{integer} \\ \coth(k_I aN)\tanh(k_I a/2) \;&=\; (1-2/\eta) \;, \end{aligned} \tag{3.15}$$

with

$$m^2 = \frac{4}{a^2}\cosh^2\left(\frac{ak_I}{2}\right) \quad . \tag{3.16}$$

It is not hard to verify that Eqs (3.13), (3.14), (3.15) and (3.16) are correct. For example, a two-by-two matrix with $a = 1$ and $\eta = 4$, has eigenvalues $m^2 = 3 \pm \sqrt{5}$. The corresponding solutions to Eqs. (3.13) and (3.15) are $k = -0.9045569$ and $k_I = -1.0612751$, which yield precisely the same results via Eqs. (3.14) and (3.16).

The parameter $\eta$ allows us to interpolate between a number of familiar limits. For example, the choice $\eta = 1$ yields the mass matrix for a gauge boson distributed among $N$ sites that receives no mass contribution from the $(N+1)^{\rm th}$ link field. This is the case, for example, for the massive $X$ and $Y$ bosons in an SU(5) moose with $N+1$ sites, in which the gauge symmetry of the last site is taken to be $\mathrm{SU}(3)_C\times\mathrm{SU}(2)_W\times\mathrm{U}(1)_Y$. In this case, Eq. (3.13) reduces to

$$\cos(ka[N+1/2]) = 0 \tag{3.17}$$

which is solved by

$$ka = \pi(2n+1)/(2N+1), \quad n = 0, 1, \dots \quad . \tag{3.18}$$

This is consistent with the results of Ref. [11], which presents the spectrum of a deconstructed 5D SU(5) theory with unified symmetry broken explicitly at an orbifold fixed point. Since we generally assume that $v_i \gg v$, and hence that $\eta \gg 1$, a more relevant limit is the one in

which $\eta \to \infty$. As we described earlier, this corresponds to a physical system in which the $N^{\text{th}}$ block has effectively become fixed. In this case, Eq. (3.13) reduces to

$$\cos(ka[N-1/2]) = 0 \tag{3.19}$$

which is solved by

$$ka = \pi(2p+1)/(2N-1), \quad p = 0, 1, \ldots \ . \tag{3.20}$$

As we expect, this result is simply Eq. (3.18) with the replacement $N \to N-1$. For large values of $N$, the expression for $m$ is approximately

$$m \approx k \approx \frac{\pi}{2Na}(2p+1) \equiv \frac{M_c}{2}(2p+1) \ , \quad p = 0, 1, \ldots \tag{3.21}$$

where we have identified the compactification scale $M_c = \pi/Na$. This is shifted relative to the gauge bosons with zero modes, which have the spectrum $m \approx M_c p$ in the same limit. This is exactly the behavior we expect in the Higgsless theory [14], obtained when when takes the boundary vevs $v_i \to \infty$. Thus, the present work demonstrates how one may obtain a four-dimensional analog for the 5D Higgsless trinified model described in Ref. [9].

With Eqs. (3.3), (3.13), (3.14), (3.15), and (3.16) in hand, we have all the information we need to take into account the effect of finite boundary vevs on the gauge boson mass spectrum. We will use this result in our study of gauge unification in the following section.

## IV. GAUGE UNIFICATION

Aside from the $\mathcal{N} = 2$ vector supermultiplets that we expect at each massive KK level of the deconstructed theory [16, 17], we assume that the only other fields relevant for unification are the light matter and Higgs fields of the MSSM. We justify this assumption in Sec. V, where we demonstrate how this can be arranged. We will find it convenient to express our results in terms of the differences

$$\delta_i(\mu) \equiv \alpha_i^{-1}(\mu) - \alpha_1^{-1}(\mu) \ , \tag{4.1}$$

where $\mu$ is the renormalization scale. Since the theory of interest to us here represents a deconstructed version of the 5D trinified theory of Ref. [9], the basic quantities of interest in studying gauge unification at the one loop level have the same form:

$$\delta_i(\mu) = \delta_i(m_H^{(1)}) - \frac{1}{2\pi} R_i(\mu) \ , \tag{4.2}$$

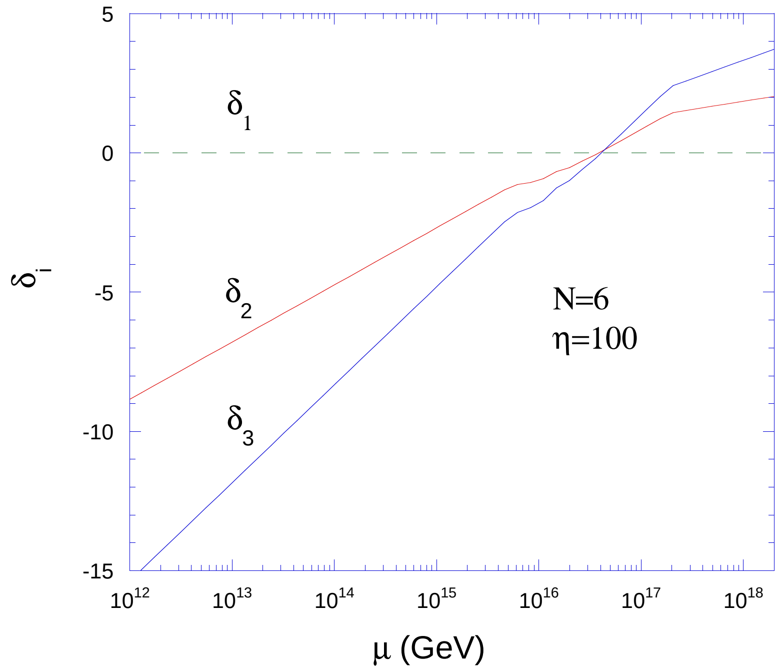


FIG. 4: Gauge unification for $N = 6$ and $\eta = 100$. See the text for details.

$$R_2(\mu) = -\frac{28}{5}\ln\left(\frac{\mu}{m_H^{(1)}}\right) - 4\sum_{0<m_0^{(n)}<\mu}\ln\left(\frac{\mu}{m_0^{(n)}}\right) + 4\sum_{0<m_H^{(n)}<\mu}\ln\left(\frac{\mu}{m_H^{(n)}}\right) \ , \tag{4.3}$$

$$R_3(\mu) = -\frac{48}{5}\ln\left(\frac{\mu}{m_H^{(1)}}\right) - 6\sum_{0<m_0^{(n)}<\mu}\ln\left(\frac{\mu}{m_0^{(n)}}\right) + 6\sum_{0<m_H^{(n)}<\mu}\ln\left(\frac{\mu}{m_H^{(n)}}\right) \ . \tag{4.4}$$

Here, $m_0^{(n)}$ and $m_H^{(n)}$ represent the mass levels corresponding to gauge fields with and without zero modes, respectively. These expressions are valid for $\mu > m_H^{(1)}$, which we assume to be the lightest gauge field KK excitation (recall that as $\eta \to \infty$, $m_H^{(1)} \to m_0^{(1)}/2$). The unification scale $M_{GUT}$ is identified with the scale at which the moose is reduced to the diagonal subgroup SU(3)$_C\times$SU(3)$_L\times$SU(3)$_R$; this is usually taken to be $2a^{-1} = 2vg$, the scale of the heaviest KK excitation. Thus we require

$$\delta_2(2a^{-1}) = 0 \ , \tag{4.5}$$

which determines the unification scale in terms of $3N$ and $\eta$. For arbitrary $\eta$, Eq. (4.5) can be solved numerically; for example, with $N = 6$ and $\eta = 100$, one finds that $M_{\rm GUT} = 2a^{-1}$ is $3.85 \times 10^{16}$ GeV, as is shown in Fig. 4. The changes in slope of $\delta_2$ and $\delta_3$ above $m_H^{(1)} \approx 0.284\, a^{-1}$ correspond to vector multiplet mass thresholds. The one kink present above the

unification scale corresponds to the mass threshold that decouples in the $\eta \to \infty$ limit. One finds that the quality of unification in this example is given by

$$\frac{\delta\alpha_3^{-1}(2a^{-1})}{\alpha_1^{-1}(2a^{-1})} \approx -0.3\%. \tag{4.6}$$

Note that the unification scale is above $2 \times 10^{16}$ GeV, the unification scale of conventional supersymmetric theories.

Since we are interested in $\eta \gg 1$, we will analyze the general case as an expansion in $1/\eta$. The results we present below may be derived by taking into account the following simplifications,

$$\sum_{0<m_0^{(n)}<\mu} \ln m_0^{(n)} = \frac{1}{2}(N-1)\ln\left(\frac{4}{a^2}\right) + \frac{1}{2}\ln\left(\frac{4N}{2^{2N}}\right) \ , \tag{4.7}$$

and

$$\sum_{0<m_H^{(n)}<\mu} \ln m_H^{(n)} = -\frac{1}{2}(N-1)\ln a^2 - \frac{1}{2\eta} + \mathcal{O}(1/\eta^2) \ , \tag{4.8}$$

which are valid if the renormalization scale is above all but the last mass threshold $m_H^{(N)}$ (*i.e.*, the kink above the unification point shown in Fig. 4 is not included in the sums). Then it follows that

$$2a^{-1} \approx 2 \times 10^{16} \text{ GeV} \times \ e^{5/(14\eta)}\, N^{5/14} \ , \tag{4.9}$$

which clarifies the previously noted delay in unification by showing the scaling in $N$ and the weak dependence on $\eta$, for $\eta$ large. One may also use Eqs. (4.7) and (4.8) to obtain a general expression for the quality of unification (*e.g.* Eq. (4.6)). One finds,

$$\frac{\delta\alpha_3^{-1}(2a^{-1})}{\alpha_1^{-1}(2a^{-1})} = -0.85\% \cdot f(N,\eta) \ , \tag{4.10}$$

where

$$f(N,\eta) = \frac{1 - 0.340(\ln N + 1/\eta)}{1 + 0.028N - 0.016\ln N + 0.004/\eta} \ . \tag{4.11}$$

The function $f$ ranges approximately from $-0.15$ to $0.97$, for large $\eta$, assuring reasonably accurate unification.

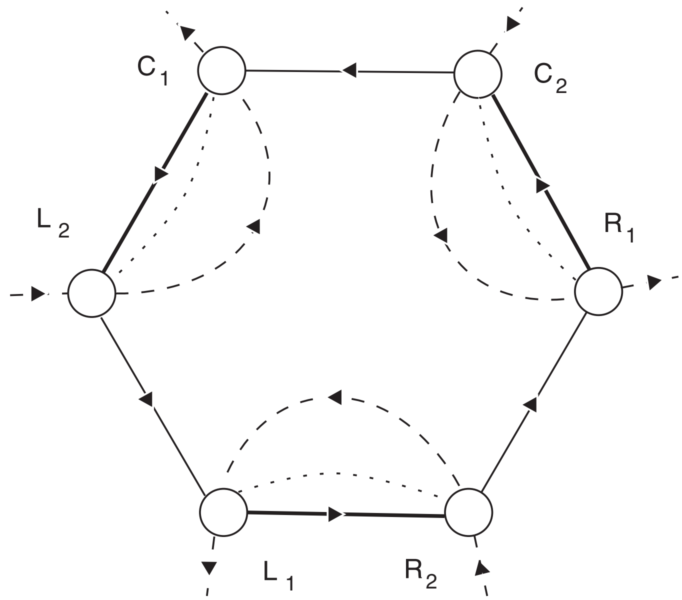


FIG. 5: Embedding of $\overline{\mathbf{27}}$ (dashed lines) and **192** (dotted lines) Higgs fields, which get vevs only at the $L_1$-$R_2$ link. Exterior lines represented spectators for anomaly cancellation.

## V. LIGHT HIGGS AND MATTER FIELDS

We now show how one may construct tri-N-ified theories with the low-energy particle content of the MSSM. Our first example assumes that the $Z_3$ symmetry applies to the theory as a whole, including the fermion representations, which makes achieving the desired low-energy theory nontrivial. Let us consider a viable model for the next-to-minimal case of $N = 2$.

We would first like to arrange for precisely two Higgs doublet fields in the low-energy spectrum. We follow the approach of Ref. [9] and consider doublets living within **27** and $\overline{\mathbf{27}}$ representations of the diagonal subgroup, and an additional Higgs field in the **192**-dimensional representation, to arrange for doublet-triplet splitting. The Higgs **27** consists of the 'special' links in the original circular moose, namely, the links between the sites $L_1$-$R_2$, $R_1$-$C_2$, and $C_1$-$L_2$. The main purpose of these links was to cancel anomalies, and to provide for the breaking of the diagonal gauge group at a boundary. We add the $\overline{\mathbf{27}}$ as shown by the dashed lines in Fig. 5. Notice that to avoid anomalies, we have added spectator fields; for example, the $(\mathbf{1}, \bar{\mathbf{3}}_1, \mathbf{3}_2)$ component requires three spectators with quantum numbers $S^a_{L_1} \sim (\mathbf{1}, \mathbf{3}_1, \mathbf{1})$, for $a = 1 \ldots 3$ and three with quantum numbers $S^a_{R_2} \sim (\mathbf{1}, \mathbf{1}, \bar{\mathbf{3}}_2)$. The links

that acquire diagonal vevs generate mass terms for these spectators,

$$W = \lambda_R^{ab} S_{R_1}^b(\mathbf{3}_1) Q_R(\bar{\mathbf{3}}_1, \mathbf{3}_2) S_{R_2}^a(\bar{\mathbf{3}}_2) + (R \to C) + (R \to L) \ , \tag{5.1}$$

where the $\lambda$ are coupling constants. While the spectators are present at the unification scale, they do not alter the renormalization group analysis presented in the previous section. With both $\mathbf{27}$ and $\overline{\mathbf{27}}$ Higgs fields present in the theory, we can arrange for heavy masses for all the color-triplet components. Doublet-triplet splitting may be obtained by adding the Higgs representation $\Omega(\mathbf{192}) \sim (\mathbf{1}, \mathbf{8}_1, \mathbf{8}_2) + (\mathbf{8}_2, 1, \mathbf{8}_1) + (\mathbf{8}_1, \mathbf{8}_2, 1)$. Notice that the representation $(\mathbf{1}, \mathbf{8}_i, \mathbf{8}_j)$ is anomaly free, so we do not require any additional spectator fields. We assume that only the $(\mathbf{1}, \mathbf{8}_1, \mathbf{8}_2)$ component of the $\mathbf{192}$ and the $(\mathbf{1}, \bar{\mathbf{3}}, \mathbf{3})$ of the $\overline{\mathbf{27}}$ acquire GUT-breaking vacuum expectation values, which again localizes the breaking of the diagonal subgroup to the $L_1$-$R_2$ link. These new vevs contribute to the the matrix $\Delta$ in Eq. (3.5), but otherwise do not alter the discussion of Section IV. In addition, appropriate vevs in the $\mathbf{192}$ and the $\overline{\mathbf{27}}$ prevent one from eliminating the desired symmetry-breaking effect of Eq. (2.2) via a gauge transformation. Calling the $\overline{\mathbf{27}}$ Higgs $\overline{Q}$, the superpotential terms relevant to doublet-triplet splitting are given by

$$W = \mu Q_{R_1C_2} \overline{Q}_{R_1C_2} + \mu Q_{C_1L_2} \overline{Q}_{C_1L_2} + Q_{L_1R_2}(\mu + h\Omega_{L_1R_2}) \overline{Q}_{L_1R_2} \ . \tag{5.2}$$

As described in Ref. [9], a vev of the form

$$\langle \Omega_{ab}^{\alpha\beta} \rangle = v_\Omega \, T^{8}{}^a_b \, T^{3}{}^\alpha_\beta \tag{5.3}$$

allows one to make a doublet component of $Q_{L_1R_2}$ and a doublet component of $\overline{Q}_{L_1R_2}$ light, provided that $\mu \approx -4\sqrt{3} h v_\Omega$. The light components, which we identify as the two Higgs doublets of the MSSM are localized at the $L_1$-$R_2$ link. Note that the down-type Higgs doublet lives within the $\overline{\mathbf{27}}$ representation, which does not have a direct Yukawa coupling with matter fields transforming in the $\mathbf{27}$. However, $\overline{Q}^2/\Lambda$ transforms as a $\mathbf{27}$, where $\Lambda$ is an ultraviolet cutoff. This combination provides for down-type Higgs Yukawa couplings via higher-dimension operators, suppressed relative to the up-type couplings by a factor of $v_i/\Lambda$.

We now must arrange for the embedding of three generations of matter fields, so that there are adequate couplings to the light, localized Higgs doublets. A possible configuration is shown in Fig. 6. Here we have chosen to embed the matter fields so that the moose

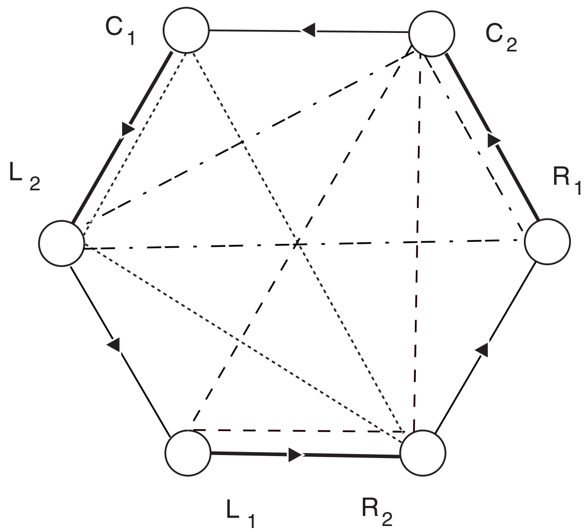


FIG. 6: Placement of three generations of matter fields in the $N = 2$ model. The dotted, dot-dashed and dashed lines correspond to the first, second and third generations, respectively.

remains invariant under 120° rotations. One could argue, therefore, that the $Z_3$ symmetry forces the existence of three generations in this model. Since the light Higgs doublets are only present on the $L_1$-$R_2$ link, while the generations are delocalized we expect that Yukawa couplings originate via higher-dimension operators, suppressed by powers of $v/\Lambda$. For the model to be successful, these suppression factors must not be so great that we are prevented from achieving phenomenologically viable Yukawa textures. Let us identify the matter **27**'s by the sites that they connect, *e.g.*, $\psi(212)$ transforms under the subset of gauge groups ($C_2$,$L_1$,$R_2$). Then we identify the three generations as follows:

$$\begin{aligned} \psi^{122} &= \text{generation 1} \\ \psi^{221} &= \text{generation 2} \\ \psi^{212} &= \text{generation 3} \;. \end{aligned} \tag{5.4}$$

Given this choice, we see, for example, that the third generation Yukawa couplings are not suppressed,

$$W \supset h_t H_u(\mathbf{1},\mathbf{3}_1,\overline{\mathbf{3}}_2)\,\psi^{212}_{RC}(\overline{\mathbf{3}}_\mathbf{2},\mathbf{1},\mathbf{3}_2)\,\psi^{212}_{CL}(\mathbf{3}_2,\overline{\mathbf{3}}_\mathbf{1},\mathbf{1}) \;, \tag{5.5}$$

while a purely second generation coupling

$$\frac{1}{\Lambda^2} h_c H_u(\mathbf{1},\mathbf{3}_1,\overline{\mathbf{3}}_2)\,\psi^{221}_{RC}(\overline{\mathbf{3}}_2,\mathbf{1},\mathbf{3}_1)\,\psi^{221}_{CL}(\mathbf{3}_2,\overline{\mathbf{3}}_2,\mathbf{1})\,Q_L(\overline{\mathbf{3}}_1,\mathbf{3}_2)\,Q_R(\overline{\mathbf{3}}_1,\mathbf{3}_2) \tag{5.6}$$

involves a suppression factor of order $v^2/\Lambda^2$. One therefore finds that the entries of the up quark Yukawa matrix involve suppression factors summarized by

$$\text{Gauge suppression factors} \sim \begin{pmatrix} \xi & \xi^2 & \xi^2 \\ \xi^3 & \xi^2 & \xi \\ \xi^2 & \xi & 1 \end{pmatrix} \quad , \tag{5.7}$$

where we have defined $\xi = v/\Lambda$; the down sector suppression factors are one power of $v_i/\Lambda$ higher. The tightest constraint on the size of these parameters comes from the strange quark Yukawa coupling, which requires that $\xi\sqrt{v_i/\Lambda} \gtrsim 1/20$. Provided that this is satisfied, one may choose coupling matrices $h_u$ and $h_d$ to obtain the additional flavor suppression required to produce Yukawa textures that are phenomenologically viable (for example, compare to those found in Ref. [18]). The reader can check that the same is true in the charged lepton sector as well.

In addition, the **27**'s of matter fields contain exotic particle content that becomes heavy in minimal trinified theories due to the SU(3)$^3$-breaking vevs of the $L_1$-$R_2$ link. The same happens here, though there is some suppression due to the delocalization of the matter multiplets. Since a generic link field transforms as $Q \sim (\overline{\mathbf{3}}_i, \mathbf{3}_{i+1})$, and it is possible to arrange $Q^2/\Lambda \sim (\mathbf{3}_i, \overline{\mathbf{3}}_{i+1})$, by elementary SU(3) group theory, one has the necessary building blocks to reproduce *any* mass operator of the $N = 1$ theory at some order in $\langle Q \rangle/\Lambda$. Note that the exotic fields in the **27** form complete SU(5) multiplets [9], so they can appear below the unification scale without substantially altering the conclusions of Section IV.

While the previous model can accommodate the flavor structure of the standard model, it is not clear whether successful models can be constructed for larger $N$. The assumption of the unbroken $Z_3$ symmetry would force different generations to be widely separated if the number of sites is large. Let us consider the possibility that the unified boundary condition on the gauge couplings has a dynamical origin and is not due to symmetry. Equivalently, we may assume that the $Z_3$ symmetry is an accidental symmetry of the gauge and Higgs sector, but not of the matter fields. Then we may place all three generations on a single set of gauge sites, namely, $C_j$-$L_1$-$R_2$, for some $j$. In this case, no suppression factors appear in the Yukawa couplings, and the origin of the fermion mass hierarchy is completely decoupled from the physics of unification. While we keep the arrangement of Higgs fields the same, we can add an additional three pairs of $\mathbf{3}$ and $\overline{\mathbf{3}}$ spectator fields at each new gauge site; in this

way, all spectators will acquire GUT-scale masses by coupling in pairs via an intermediate link field. For either the $C$, $L$ or $R$-sector spectators, these interactions have the form

$$W = \sum_{i=1}^{N-1} \lambda_i^{ab} S_i^a(\mathbf{3}_i)\, Q(\overline{\mathbf{3}}_i, \mathbf{3}_{i+1})\, S_{i+1}^b(\overline{\mathbf{3}}_{i+1}) \quad . \tag{5.8}$$

Using this construction, we can study models with many more gauge sites. The only limitation on the size of $N$ comes from the assumption that the SU(3) gauge couplings $g$ remain perturbative. For the case of identical gauge couplings, $g$ is related to the gauge coupling of the diagonal subgroup, $g_{\rm GUT}$, by

$$g = \sqrt{N} g_{\rm GUT} \quad , \tag{5.9}$$

Using our earlier renormalization group analysis, one finds numerically that the constraint $g^2/(4\pi) < 1$ translates to

$$N \leq 65 \quad . \tag{5.10}$$

## VI. CONCLUSIONS

We have considered a natural generalization of trinification to theories with $3N$ gauge groups. We have showed that the gauge and Higgs sector of the theory can be interpreted as a deconstructed version of a 5D trinified theory with unified symmetry broken at a boundary; we have studied the gauge spectrum in the deconstructed theory for arbitrary boundary Higgs vevs. As in the 5D theory, the differences in inverse gauge couplings evolve logarithmically, and one finds successful unification at a scale higher than in the 4D MSSM. Since the unification scale grows as $N^{5/14}$, unification may be delayed to $9 \times 10^{16}$ GeV before the gauge coupling of the replicated SU(3) factors becomes nonperturbative. This might be compared to the result in the 5D theory, $1 \times 10^{17}$ GeV, which is the point at which the 5D Planck scale and the unification scale coincide [9].

We have also considered ways in which the light matter and Higgs content of the MSSM could be embedded in the (rather large) unified group. We first showed how doublet-triplet splitting could be arranged, via fine tuning, leaving light Higgs doublets localized between two gauge sites in the original moose. The point here is not to solve the doublet-triplet splitting problem, but to present a viable zeroth-order framework in which such questions can be studied. With all symmetry breaking localized, we then considered the embedding

of matter fields. Assuming first that the unified boundary condition on the gauge couplings arises from a $Z_3$ symmetry that rotates the moose by 120°, one obtains symmetrical embeddings of the fermion multiplets within the moose (*c.f.*, Fig. 6). Although these have no strict extra-dimensional interpretation, we have, roughly speaking, delocalized the fermion multiplets while keeping all the symmetry-breaking and light Higgs fields at one point. We show for relatively small moose, one can arrange for sufficient overlaps via higher-dimension operators involving the link fields to account for fermion masses and inter-generational mixing. For larger moose, we assume instead that the unified boundary condition on the gauge couplings is simply present (or alternately, the $Z_3$ symmetry is an accidental symmetry of only the gauge and Higgs sectors) so that matter can be embedded asymmetrically. Then the overlap between matter fields and light Higgs fields can be made maximal and no restriction follows on the size of the moose. Alternatively, such models provide the freedom to adjust matter-matter and matter-Higgs overlaps in a way that can account in part for the flavor hierarchies of the standard model. This is one advantage of the models introduced here over minimal trinified scenarios.

Aside from demonstrating how one may deconstruct the 5D trinified theory described in Ref. [9], we have arrived at at class of 4D theories that are interesting in their own right. This is not a surprise, as the same is true of deconstructed 5D theories based on SU(5) symmetry that have been studied in the literature [11]. The present work therefore provides a new framework for future study of a range of familiar issues, including the implementation of solutions to the doublet-triplet splitting problem, proton decay phenomenology, the origin of flavor via combined unified and horizontal symmetries, construction of explicit symmetry breaking sectors, *etc.* Models based on the idea of nonsupersymmetric triplicated trinification [5] may also be worthy of study in this context.

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